ANSWER KEY

NATIONAL INSTITUTE OF TECHNICAL TEACHERS TRAINING AND RESEARCH

(DEEMED TO BE UNIVERSITY UNDER DISTINCT CATEGORY)

CHANDIGARH

Ph.D. Entrance Examination 2024

Subject / Branch / Department	:	APPLIED SCIENCE (MATHEMATICS)
Roll No.	:	
Candidate Name	:	
Date of Examination	:	

Maximum Marks: 25 (There is no negative marking)

Notes: (a) Only one option to be tick-marked out of the four options given as answer

(b) The Candidate must put his/her signature with date at the bottom of each page

(c) For any rough work, please use ONLY back-sides of pages which are left blank

1. The Charpit's equation for the PDE $up^2+q^2+x+y=0, p=\frac{\partial u}{\partial x}, q=\frac{\partial u}{\partial y}$ are given by:

(A) $\frac{dx}{-1-p^2}=\frac{dy}{-1-qp^2}=\frac{du}{2p^2u+2q^2}=\frac{dp}{2pu}=\frac{dq}{2q}$

(B) $\frac{dx}{2pu}=\frac{dy}{2q}=\frac{du}{2p^2u+2q^2}=\frac{dp}{-1-p^3}=\frac{dq}{-1-qp^2}$

(C) $\frac{dx}{up^2}=\frac{dy}{q^2}=\frac{du}{0}=\frac{dp}{x}=\frac{dq}{y}$

(D) $\frac{dx}{2q}=\frac{dy}{2pu}=\frac{du}{x+y}=\frac{dp}{p^2}=\frac{dq}{qp^2}$

2. Consider the Linear Programming problem:

$Minimize\ z=-2x-5y, subject\ to\ 3x+4y\geq 5, x\geq 0, y\geq 0$

Which of the following is correct ?

(A) Set of feasible solution is empty

(B) Set of feasible solutions is non empty but there is no optimal solution

(C) Optimal value is attained at (0, 5/4)

(D) Optimal value is attained at (5/3, 0)

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3. Hundred (100) tickets are marked as 1,2,….., 100 and are arranged at random. Four tickets are picked from these tickets and are given to four persons A, B, C and D. What is the probability that A gets the ticket with the largest value (among A,B,C,D) and D gets the ticket with the smallest value (among A, B,C, D) ?

(A) 1/4

(B) 1/6

(C) 1/2

(D) 1/12

4. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\theta,1)$, where $\theta \in \{1,2\}$. Then which of the following statements about the maximum likelihood estimator (MLE) of θ is correct –

(A) MLE of θ does not exist

(B) MLE of θ is $\overline{X}$

(C) MLE of θ exists but it is not $\overline{X}$

(D) MLE of θ is an unbiased estimator of θ

5. The matrix $\begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$ is –

(A) Positive definite

(B) Non-negative definite but not positive definite

(C) Negative definite

(D) Neither negative definite nor positive definite

6. A group G is generated by the element x, y with the relations $x^3 = y^2 = (xy)^2 = 1$, the order of G is

(A) 4

(B) 6

(C) 8

(D) 12

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7. The resolvent kernel $R(x,t,\lambda)$ for the Volterra integral equation $\phi(x) = x + \lambda\int_a^x \phi(s)ds$ is

(A) $e^{\lambda(x+t)}$

(B) $e^{\lambda(x-t)}$

(C) $\lambda e^{(x+t)}$

(D) $e^{\lambda xt}$

8. Consider the M/M/1 queue with the arrival rate λ and service rate μ with $\mu > \lambda$. What is the probability that no customer exited the system before time 5 ?

(A) $\dfrac{\mu e^{-5\lambda} - \lambda e^{-5\mu}}{\mu - \lambda}$

(B) $e^{-5\lambda} - e^{-5\mu}$

(C) $e^{-5\lambda} + (1 - e^{-5\lambda})\dfrac{e^{-5\mu}}{5\mu}$

(D) $e^{-5\mu} + (1 - e^{-5\mu})\dfrac{e^{-5\lambda}}{5\lambda}$

9. Consider the power series $f(x) = \sum_{n=2}^{\infty} \log(n) x^n$. The radius of convergence of the series $f(x)$ is –

(A) 0

(B) 1

(C) 3

(D) ∞

10. Two students are solving the same problem independently. If the probability that the first solves the problem is 3/5 and the probability that the second solves the problem is 4/5 , what is the probability that the at least one of the them solves the problem ?

(A) 17/25

(B) 19/25

(C) 21/25

(D) 23/25

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11. Let $W_1 = \{(u,v,w,x) \in R^4, u+v+w=0, 2v+x=0, 2u+2w-x=0\}$ and

$W_2 = \{(u,v,w,x) \in R^4, u+w+x=0, u+w-2x=0, v-x=0\}$. Then which among the following is true ?

(A) $\dim(W_1) = 1$

(B) $\dim(W_2) = 2$

(C) $\dim(W_1 \cap W_2) = 1$

(D) $\dim(W_1 + W_2) = 3$

12. Let *f(x)* be a polynomial of unknown degree taking the values –

x	0	1	2	3
F(x)	2	7	13	16

All the fourth divided differences are -1/6. Then the coefficient of x^3 is –

(A) 1/3

(B) -2/3

(C) 16

(D) -1

13. Let V denote the vector space of real valued continuous functions on the closed interval [0, 1]. Let W be the sub space of V spanned by $\{\sin(x), \cos(x), \tan(x)\}$. Then the dimension of W over R is –

(A) 1

(B) 2

(C) 3

(D) Infinite

14. The maximum value of objective function $Z = 5x_1 + 2x_2$ under the linear constraints $x_1, x_2 \geq 0, x_1 \geq x_2, \quad 2 \leq x_1 + x_2 \leq 4$ is –

(A) 14

(B) 20

(C) 25

(D) 27

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15. Consider the following LPP:

$$Max.Z = x_1 + \frac{5}{2}x_2,$$

Subject to $5x_1 + 3x_2 \leq 15,$

$-x_1 + x_2 \leq 1,$

$2x_1 + 5x_2 \leq 10,\ x_1, x_2 \geq 0$

Then the problem –

(A) Has no feasible solution

(B) Has indefinitely many optimal solutions

(C) Has an unique optimal solution

(D) Has an unbounded solution

16. The value of $\lim\limits_{z \to 2e^{i\pi/3}} \dfrac{z^3 + 8}{z^4 + 4z^2 + 16}$ is –

(A) $\frac{3}{8}(1+i)$

(B) $\frac{2}{3}(1+i)$

(C) $\frac{1}{8}(1-i)$

(D) $\frac{3}{8}(1-i)$

17. Customers arrives at a sales counter managed by a single person according to Poisson process with a mean rate of 20 per hour. The time required to serve a customer has an exponential distribution with a mean if 100 seconds. What is the average waiting time of a customer –

(A) 225

(B) 200

(C) 250

(D) 150

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18. The objective function of the dual problem for the following primal LPP:

$Max. f = 2x_1 + x_2,$
Subject to $x_1 - 2x_2 \geq 2,$
$x_1 + 2x_2 = 8,$
$x_1 - x_2 \leq 11,$

With $x_1 \geq 0$ and x_2 unrestricted in sign, is given by

(A) $Min. z = 2y_1 - 8y_2 + 11y_3$

(B) $Min. z = 2y_1 + 8y_2 + 11y_3$

(C) $Min. z = 2y_1 - 8y_2 - 11y_3$

(D) $Min. z = 2y_1 + 8y_2 - 11y_3$

19. Men arrive in a queue according to a Poisson process with rate λ_1 and women arrive in the same queue according to another Poisson process with rate λ_2. The arrivals of men and women are independent. The probability that the first arrival in the queue is a man, is –

(A) $\frac{\lambda_1}{\lambda_1 + \lambda_2}$

(B) $\frac{\lambda_2}{\lambda_1 + \lambda_2}$

(C) $\frac{\lambda_1}{\lambda_2}$

(D) $\frac{\lambda_2}{\lambda_1}$

20. The value of integral $\int_{|z|=2} \frac{e^{2z}}{(z+1)^4} dz$

(A) $2\pi i e^{-1}$

(B) $\frac{8}{3}\pi i e^{-2}$

(C) $\frac{2}{3}\pi i e^{-2}$

(D) 0

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21. The coefficient of z^{-3} in the Laurent's expansion of the function $f(z) = \frac{1}{(z+1)(z+3)}$ in the domain $|z| > 3$ is:

(A) 8

(B) -8

(C) -4

(D) 4

22. The probabilities of X, Y and Z becoming managers are 4/9, 2/9 and 1/3 respectively. The probabilities that the Bonus scheme will be introduced if X, Y and Z becomes managers are 3/10, ½ and 4/5 respectively. Then, what is the probability that Bonus scheme will be introduced?

(A) 23/45

(B) 24/90

(C) 12/45

(D) 23/90

23. The joint *p.d.f* of a two-dimensional random variable *(X, Y)* is given by:

$$f(x,y) = \begin{cases} 2, 0 < x < 1, 0 < y < x; \\ 0, \textit{elsewhere} \end{cases}$$

The marginal density function of X and Y is –

(A) 2(1 – y)

(B) 2(1+y)

(C) 1-y

(D) 1+2y

24. Let A be 5×5 matrix and let B be obtained by changing one element of A. Let r and s be the ranks of A and B respectively. Which of the following statements is correct –

(A) $s \leq r + 1$

(B) $r \leq s$

(C) $s = r - 1$

(D) $s \neq r$

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25. Consider the integral equation $y(x) = x^3 + \int_0^x \sin(x-t)y(t)dt,\ x \in [0, \pi]$. Then the value of y(1) is-

(A) 19/20

(B) 1

(C) 17/20

(D) 21/20

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